5/4/18 "To err is human; to forgive, divine." - Alexander Pope

HW: "Basic Graphs of Sine and Cosine" homework section

AIM: How do we graph Sine and Cosine functions?

Warm Up:

Fill in the Unit Circle

FLUENCY

1. Determine the value of each of the following in exact and simplest form (leave no complex fractions).

(a) $\csc(30^\circ)$

$$\boxed{=\frac{1}{\sin(30^\circ)}=\frac{1}{\frac{1}{2}}=1\cdot\frac{2}{1}=2}$$

(b) $\cot(90^\circ)$

$$\boxed{=\frac{\cos(90^\circ)}{\sin(90^\circ)}=\frac{0}{1}=0}$$

(c) $\sec(180^\circ)$

$$\boxed{=\frac{1}{\cos(180^\circ)}=\frac{1}{-1}=-1}$$

(d) $\cot\left(\frac{\pi}{3}\right)$

$$\boxed{\begin{array}{l}=\dfrac{\cos\left(\frac{\pi}{3}\right)}{\sin\left(\frac{\pi}{3}\right)}=\dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}=\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\sqrt{3}}\\ =\dfrac{1}{\sqrt{3}}\ \text{or}\ \dfrac{\sqrt{3}}{3}\end{array}}$$

(e) $\csc\left(\frac{3\pi}{2}\right)$

$$\boxed{=\frac{1}{\sin\left(\frac{3\pi}{2}\right)}=\frac{1}{-1}=-1}$$

(f) $\sec\left(\frac{5\pi}{4}\right)$

$$\boxed{\begin{array}{l}=\dfrac{1}{\cos\left(\frac{5\pi}{4}\right)}=\dfrac{1}{-\frac{\sqrt{2}}{2}}=1\cdot-\frac{2}{\sqrt{2}}\\ =-\dfrac{2}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=-\dfrac{\cancel{2}\sqrt{2}}{\cancel{2}}=-\sqrt{2}\end{array}}$$

2. Use your calculator to determine the value of each of the following to the nearest *hundredth*.

(a) $\cot(115^\circ)$

$$\boxed{\begin{array}{l}=\dfrac{\cos(115^\circ)}{\sin(115^\circ)}=-0.466...\\ \approx -0.47\end{array}}$$

(b) $\sec(312^\circ)$

$$\boxed{\begin{array}{l}=\dfrac{1}{\cos(312^\circ)}=1.494...\\ \approx 1.49\end{array}}$$

(c) $\csc(245^\circ)$

$$\boxed{\begin{array}{l}=\dfrac{1}{\sin(245^\circ)}=-1.103...\\ \approx -1.10\end{array}}$$

3. In simplest radical form, $\sec(135^\circ)$ is equal to

(1) $-\frac{\sqrt{2}}{3}$ (3) $-\frac{\sqrt{2}}{2}$

(2) $-\sqrt{2}$ (4) $-\frac{\sqrt{3}}{2}$

$$\boxed{\begin{array}{l}=\dfrac{1}{\cos(135^\circ)}=\dfrac{1}{-\frac{\sqrt{2}}{2}}=-\frac{2}{\sqrt{2}}\\ =-\dfrac{2}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}=-\dfrac{\cancel{2}\sqrt{2}}{\cancel{2}}=-\sqrt{2}\end{array}}$$

$\boxed{(2)}$

4. Which of the following is nearest to the value of $\cot(220^\circ)$?

(1) 1.19 (3) −2.74

(2) 3.17 (4) −0.85

$$\boxed{=\frac{\cos(220^\circ)}{\sin(220^\circ)}=1.191...\approx 1.19}$$

$\boxed{(1)}$

5. For which of the following values of α is $\cot(\alpha)$ undefined?

(1) 60° (3) 180°

(2) 90° (4) 135°

$$\boxed{\cot(\alpha)=\frac{\cos(\alpha)}{\sin(\alpha)} \Rightarrow \sin(\alpha)=0 \quad \alpha=180^\circ}$$

$\boxed{(3)}$

6. For which angle, β, below will $\sec(\beta)$ not exist?

(1) 30° (3) 180°

(2) 45° (4) 90°

$$\boxed{\sec(\beta)=\frac{1}{\cos(\beta)} \Rightarrow \cos(\beta)=0 \quad \beta=90^\circ}$$

$\boxed{(4)}$

7. Determine whether each function in the tables below is positive, $(+)$, or negative, $(-)$, for angles whose terminal rays lie in the respective quadrants. Use the table in part (a) to help create the table in (b).

(a)

	I	II	III	IV
$\cos(\theta)$	(+)	(–)	(–)	(+)
$\sin(\theta)$	(+)	(+)	(–)	(–)

(b)

	I	II	III	IV
$\tan(\theta)$	(+)	(–)	(+)	(–)
$\cot(\theta)$	(+)	(–)	(+)	(–)
$\sec(\theta)$	(+)	(–)	(–)	(+)
$\csc(\theta)$	(+)	(+)	(–)	(–)

8. For the angle β it is known that $\csc(\beta)>0$ and $\sec(\beta)<0$. When drawn in standard position, the terminal ray of β lies in quadrant

(1) I (3) III

(2) II (4) IV

$$\boxed{\csc(\beta)>0 \Rightarrow \text{I or II} \quad \sec(\beta)<0 \Rightarrow \text{II or III} \quad \text{II}}$$

$\boxed{(2)}$

9. The angle θ when drawn in standard position has its terminal ray in the second quadrant. If it is known that $\sin\theta=\frac{5}{13}$ then determine the values of all of the remaining trigonometric functions.

(a) $\cos\theta$

$$\boxed{\sin^2\theta+\cos^2\theta=1 \Rightarrow \left(\frac{5}{13}\right)^2+\cos^2\theta=1 \quad \cos^2\theta=\frac{144}{169} \Rightarrow \cos\theta=\pm\sqrt{\frac{144}{169}} \Rightarrow -\frac{12}{13}}$$

(b) $\tan\theta$

$$\boxed{=\frac{\sin\theta}{\cos\theta}=\frac{5/13}{-12/13}=-\frac{5}{12}}$$

(c) $\sec\theta$

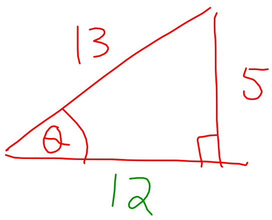
$$\boxed{=\frac{1}{\cos\theta}=\frac{1}{-12/13}=-\frac{13}{12}}$$

(d) $\csc\theta$

$$\boxed{=\frac{1}{\sin\theta}=\frac{1}{5/13}=\frac{13}{5}}$$

(e) $\cot\theta$

$$\boxed{=\frac{1}{\tan\theta}=\frac{1}{-5/12}=-\frac{12}{5}}$$



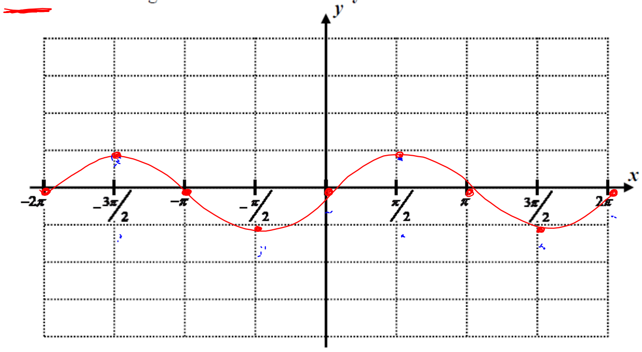
$\sin = \frac{O}{H}$ $\csc = \frac{H}{O}$

***Exercise* #1:** Consider the functions $f(x)=\sin(x)$ and $g(x)=\cos(x)$, where x is an angle in radians.

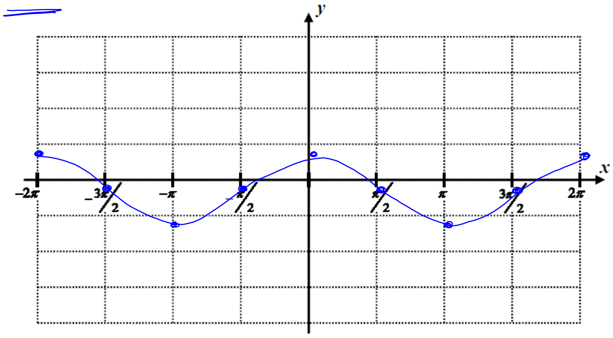
(a) By using the unit circle, fill out the table below for selected quadrantal angles.

x	-2π (−360)	$-3\pi/2$ (−270)	$-\pi$ (−180)	$-\pi/2$ (−90)	0 (0)	$\pi/2$ (90)	π (180)	$3\pi/2$ (270)	2π (360)
$\cos(x)$	1	0	−1	0	1	0	−1	0	1
$\sin(x)$	0	1	0	−1	0	1	0	−1	0

(b) Graph the sine curve on the grid shown below. **Clearly** label which curve is which.



(c) Graph the cosine curve on the grid shown below. **Clearly** label the curve.



(d) The domain and range of the sine and cosine functions are the same. State them below in interval notation.

Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

(e) After how much horizontal distance will both sine and cosine repeat its basic pattern? This is called the **period** of the trigonometric graph. Because these graphs have patterns that repeat they are called **periodic**.

2π

Now we would like to explore the effect of changing the coefficient of the trigonometric function. In essence we would like to look at the graphs of functions of the forms:

$$y = A\sin(x) \quad \text{and} \quad y = A\cos(x)$$

A = amplitude (How high/low the graph goes)

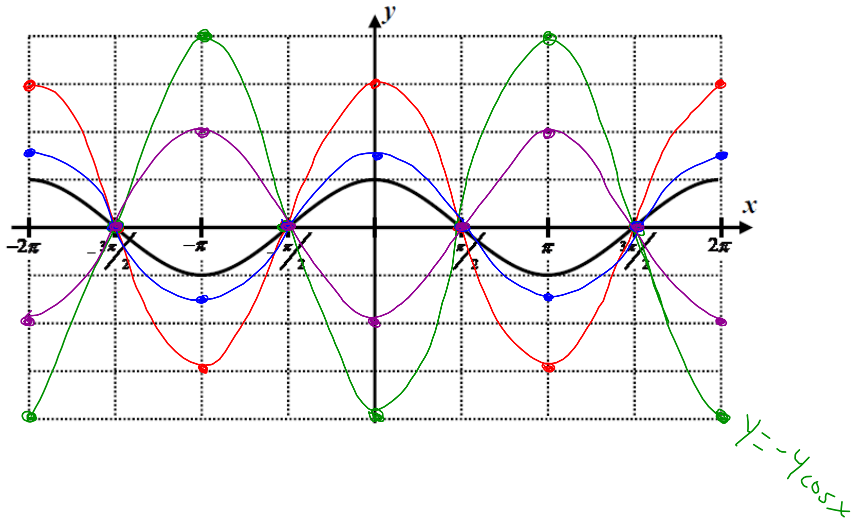
Exercise #2: The grid below shows the graph of $y = \cos(x)$. Use your graphing calculator to sketch and label each of the following equations. Be sure your calculator is in **RADIAN MODE**.

$y = 3\cos(x)$

$y = -4\cos(x)$

$y = \frac{3}{2}\cos(x)$

$y = -2\cos(x)$



As we can see, this coefficient controls the height that the cosine curves rises and falls above the *x*-axis. Its absolute value is given the name **amplitude**. In terms of sound waves it indicates the volume of the sound.

Exercise #4: The basic sine function is graphed below. **Without** the use of your calculator, sketch each of the following sine curves on the axes below.

$y = 2\sin(x)$

$y = 4\sin(x)$

$y = -3\sin(x)$

$y = -\frac{1}{2}\sin(x)$